

Review for Test II

For full credit: use calculus to solve problems, circle answers, and **show all your work**.

1) Find all critical numbers of:

$$f(x) = x^2(x + 5) \text{ using calculus.}$$

2) Find the value of c to match the mean

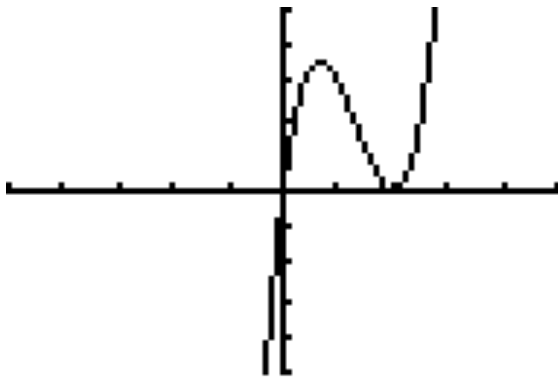
value on $[-8, 8]$ of $f(x) = x^{-\frac{1}{3}}$.

Hint: mean value theorem!

3) Find the value of c to match the mean value on $[0, \pi]$ of $f(x) = \cos x$.

4) Find all maxima and minima on $[0, 10]$ of $f(t) = t\sqrt{9-t}$ using calculus.

5) Below is a picture of a function. Sketch the graph of this function's derivative on the same axes.



6) Find all intervals on which the function $f(x) = (x - 2)^2(x - 2)$ is increasing, decreasing, and all relative extrema.

Increasing:

Decreasing:

Extrema:

Max:

Min:

7) Find the points of inflection and discuss the concavity of $f(x) = x^3 - 6x^2 + 12x$.

8) Find the limit as x approaches infinity for $f(x) = \frac{3x^2 - 8x}{x^2 + 2}$.

9) Find two positive numbers such that the second number is the reciprocal of the first and the sum is a minimum.

10) Find the length and width of a rectangle with minimum perimeter and area of 100 sq. ft.

11) Suppose a rancher buys 1200 yards of fencing to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?

12) Use Newton's method to find the zero in $f(x) = x^3 + x + 1$. The formula Newton found was: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. Show each iteration.

13) Krusty the Clown is shot out of a circus cannon and his path can be modeled by the function $f(t) = 5t^3 - 20t^2 + 20t$ where t is the time in seconds and $f(t)$ is Krusty's height in meters for the first three seconds of his flight. Find Krusty's maximum and minimum heights.

14) Write a story for the three seconds of Krusty's Crazy Kannon show. Include discussions of extrema, concavity, and inflection points.

15) Use the limit process to find the area under the curve of $y = x^2 + 2$ on $[2, 5]$.

16) Given Newton's Method generates the next guess for a zero as: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, calculate three iterations for the zero of $y = 8x^5 + 3x - 2$ using the initial guess of 1.

17) Find all critical numbers and asymptotes of the function $f(x) = (x + 1)^2(x - 5)$

18) Find all intervals on which the function $f(x) = (x + 1)^2(x - 5)$ is increasing, decreasing, and all relative extrema.

Increasing:

Decreasing:

Extrema:

Max:

Min:

19) A “Norman window” is a rectangle with a semicircle on top. Find the dimensions of a Norman window with perimeter of 16 ft and a maximum area.

20) A rectangle (in the 1st quadrant) is bounded by the x-axis, y-axis, and line $y = -0.5x + 3$. Find the dimensions of the rectangle with maximum area.

21) Use $a(t) = -9.8$ m/sec./sec. due to gravity. Find an equation to represent the velocity and another equation to represent the height of an object using v_0 and h_0 for initial velocity and height.

22) A rectangular solid with a square base has a surface area of 337.5 square cm. Find the dimensions of the rectangle to maximize the volume.

Velocity equation:

Height equation: